Soft modes in electronic stripe phases and their consequences for thermodynamics and transport

T. R. Kirkpatrick[1] and D. Belitz[2]
[1]*Department of Physics and Institute for Physical Science and Technology, University of Maryland, College Park, Maryland 20742, USA*
[2]*Department of Physics, Institute of Theoretical Science, and Materials Science Institute, University of Oregon, Eugene, Oregon 97403, USA*
(Received 27 July 2009; published 26 August 2009)

The Goldstone mode due to stripe or unidirectional charge-density-wave order in electron systems is found to have the same functional form as the one in classical smectic liquid crystals. It is very similar to the Goldstone mode that results from helical magnetic order. This allows for an effective theory that provides a quasiparticle description of either stripe phases or helimagnets in the low-energy regime. The most remarkable observable consequence is an electronic relaxation rate in $d=2$, that is, $1/\tau \propto T \ln T$ in clean systems and $1/\tau \propto \sqrt{T}$ in weakly disordered ones. The corresponding results in $d=3$ are $1/\tau \propto T^{3/2}$ and $1/\tau \propto T$, respectively.

DOI: 10.1103/PhysRevB.80.075121 PACS number(s): 71.45.Lr, 71.27.+a

I. INTRODUCTION

Classical liquid crystals display a fascinating variety of ordered phases.[1,2] A fluid of prolate molecules ("directors") can enter a nematic phase that breaks rotational invariance by aligning, on average, the major axes of the molecules while their center-of-mass motion remains fluidlike. If, in addition, the molecules arrange in layers that break translational invariance in one direction, a smectic phase results. Smectic order is characterized by a vector $\boldsymbol{q}$ whose direction is normal to the layers and whose modulus q determines the interlayer distance $2\pi/q$. One further distinguishes between smectic-A phases, where, in the ground state, the major axes of the directors are aligned perpendicular to the layers, and smectic-C phases, where they are aligned at an angle. If chiral molecules are added to the fluid, a cholesteric phase can result where translational invariance is broken by means of a helical arrangements of the directors. These various instances of spontaneously broken continuous symmetries result in the existence of Goldstone modes.[3,4] In the nematic phase the Goldstone modes correspond to uniform rotations of all molecules, analogous to the Goldstone modes in a ferromagnet. In the smectic and cholesteric phases the Goldstone mode takes the form of a propagating wave with a highly anisotropic frequency wave-vector relation

$$\Omega_s(\boldsymbol{k}) = \frac{|\boldsymbol{k}_\perp|}{|\boldsymbol{k}|}\sqrt{c_x k_x^2 + c_\perp \boldsymbol{k}_\perp^4/q^2}. \tag{1.1}$$

Here $\boldsymbol{k}=(k_x,\boldsymbol{k}_\perp)$ is the wave vector and we have chosen $\boldsymbol{q}$ to point in the x direction. c_x and $c_\perp$ are elastic constants. The excitation with frequency Ω_s is often referred to as "second sound." Ordinary, or first, sound also exists and is slightly modified by the existence of the smectic or cholesteric order. The absence of a term proportional to $\boldsymbol{k}_\perp^2$ under the square root in Eq. (1.1) is due to rotational invariance, and the functional form of $\Omega_s(\boldsymbol{k})$ is the same in both smectic and cholesteric phases.

In recent years, electronic analogs of liquid-crystal-ordered phases have been discussed in the context of quantum-Hall systems,[5–7] high-T_c superconductors,[8] and helical magnets.[9–11] An electronic nematic phase can result from an attractive electron-electron interaction in the quadrupole or $\ell=2$ channel.[12] If the interaction amplitude exceeds a critical strength, a Pomeranchuk instability[13] results in a Fermi surface that, for nearly-free electrons, is an ellipse (in $d=2$) or an ellipsoid (in $d=3$) instead of a circle or a sphere. With increasing correlation strength, a unidirectional charge-density wave can form that is the electronic analog of a smectic phase. Such one-dimensional order is normally unstable but it is stabilized by the underlying nematic order.[14] The resulting "stripe phases" are believed to be realized in quantum-Hall systems and in high-T_c superconductors.[15] An electronic analog of cholesteric order is provided by helical magnets, such as MnSi or FeGe, where the magnetization orders in a helical pattern.[16] The Goldstone mode in the latter ("helimagnon") turns out to be very similar to that in either classical cholesterics or smectics; it is given by Eq. (1.1) without the $|\boldsymbol{k}_\perp|/|\boldsymbol{k}|$ prefactor.[17] This difference is due to differences in the kinetic equations that govern the dynamics of spins and directors, respectively.

In this paper we investigate stripe order, with a focus on 2-d or quasi-2-d systems, although the corresponding 3-d results can be readily obtained and are also given. We determine the resulting Goldstone mode and its consequences for observables. We will focus on systems in a vanishing magnetic field; for discussions of soft fluctuations in stripe phases of quantum-Hall systems, see, Refs. 18–20. We find that the contribution of the Goldstone mode to the specific heat is proportional to $T^{3/2}$ and thus subleading to the Fermi-liquid contribution. The single-particle relaxation rate $1/\tau$ averaged over the Fermi surface, however, is found to go as $T \ln T$, which is a much stronger T dependence than in a Fermi liquid. The former result is consistent with the one obtained before in Ref. 21 but the latter is not. We will explain the origin of this discrepancy. In addition, our results are the $d=2$ analogs of our previous results for helimagnets.[17,22] Our result for $1/\tau$ implies an inverse thermal conductivity that goes as $T \ln T$ and we will discuss consequences for the electrical conductivity.

II. STRIPE ORDER

A. Statics

Let us assume a phase with stripe order, i.e., an electron density ρ in the ground state that can be written, in a saddle-point approximation,[23] as

$$\rho_{sp}(x) = \rho_0 + \Delta \cos(\boldsymbol{q}\cdot\boldsymbol{x}). \tag{2.1a}$$

Here ρ_0 is the average density and Δ is the amplitude of the density wave, which is the order parameter of the smectic order. $x=(\boldsymbol{x},\tau)$ comprises the spatial position $\boldsymbol{x}$ and the imaginary time τ. The density wave vector $\boldsymbol{q}$ with modulus $q\equiv|\boldsymbol{q}|$ is determined by the microscopic mechanism that causes the smectic order, e.g., a density-density correlation function that has a maximum at $q\neq 0$. In general, in an electronic smectic one expects q to be a sizable fraction of the Fermi wave number k_F.

Fluctuations about the saddle point will include amplitude fluctuations, which are massive and can be neglected, and phase fluctuations that will be soft. The fluctuating density will thus read

$$\rho(x) = \rho_0 + \Delta \cos[\boldsymbol{q}\cdot\boldsymbol{x} + u(x)], \tag{2.1b}$$

with a phase $u(x)$. The functional form of the static phase-phase correlation function is determined by rotational symmetry and must be the same as in the classical case,[2]

$$\langle u(k)u(-k)\rangle_{i\Omega=0} = \frac{1}{N_F}\frac{1}{c_x k_x^2 + c_\perp \boldsymbol{k}_\perp^4/q^2}. \tag{2.2}$$

Here $k=(\boldsymbol{k},i\Omega)$ comprises the wave vector $\boldsymbol{k}$ and a bosonic imaginary frequency $i\Omega$. Since any Gaussian action must be quadratic in Δ, the elastic constants will be proportional to Δ^2 and proportional to one another: $c_x \propto c_\perp \propto \lambda^2/k_F^2$. Here $\lambda=\Gamma\Delta$, with Γ an appropriate density-density interaction strength, is an energy that is the charge-density analog of the Stoner gap in ferromagnetism, and the density of states on the Fermi surface, N_F, serves as a normalization factor.

B. Dynamics

We now use time-dependent Ginzburg-Landau theory (TDGL) (Ref. 24) to determine the dynamics of the phase fluctuations. For density fluctuations, the appropriate kinetic equation is a Langevin equation[25]

$$\rho\partial_t \boldsymbol{v} = -\boldsymbol{\nabla}p - \rho(\boldsymbol{v}\cdot\boldsymbol{\nabla})\boldsymbol{v} - e(\rho/m_e)\boldsymbol{E} + \boldsymbol{\zeta}, \tag{2.3a}$$

augmented by the continuity equation

$$\partial_t \rho = -\boldsymbol{\nabla}\cdot(\rho\boldsymbol{v}), \tag{2.3b}$$

and the Maxwell equations[26]

$$\boldsymbol{\nabla}\cdot\boldsymbol{E} = -2^{d-1}\pi e(-\boldsymbol{\nabla}^2)^{(3-d)/2}(\rho-\rho_0), \tag{2.3c}$$

$$\boldsymbol{\nabla}\times\boldsymbol{E} = 0. \tag{2.3d}$$

Here t denotes real time, ρ is the mass density, $\boldsymbol{v}$ is the velocity, m_e and e are the electron mass and charge, respectively, $\boldsymbol{E}$ is the electric field, and $\boldsymbol{\zeta}$ is a Langevin force. The pressure p can be written as $p=(\rho/V)\partial F/\partial\rho$, with V the system volume and F the free energy. The latter is given by a Hamiltonian $H[u]$ that generates the static correlation function given by Eq. (2.2). At this point we need to realize that, because of the conserved nature of the density, fluctuations near $\boldsymbol{k}=0$ are as important as those near $\boldsymbol{k}=\boldsymbol{q}$. We thus write, to linear order in the fluctuations,

$$\rho(\boldsymbol{x}) = \rho_0 + n(\boldsymbol{x}) + \Delta\cos(\boldsymbol{q}\cdot\boldsymbol{x}) - u(\boldsymbol{x})\Delta\sin(\boldsymbol{q}\cdot\boldsymbol{x}), \tag{2.4a}$$

and

$$H[n,u] = (c_0^2/2\rho_0)\int d\boldsymbol{x}[n(\boldsymbol{x})]^2 + N_F\int d\boldsymbol{x}\{c_x[\partial_x u(\boldsymbol{x})]^2 + c_\perp[\boldsymbol{\nabla}_\perp^2 u(\boldsymbol{x})]^2\}, \tag{2.4b}$$

with c_0 the speed of (first) sound. Equation (2.3) is now fully specified and we solve them in a zero-loop approximation, which amounts to neglecting the terms nonlinear in the velocity.[27] Within TDGL, the static fields entering the Hamiltonian are replaced by the corresponding dynamic ones after performing the appropriate functional derivatives.

We now course grain the equations, i.e., we perform a spatial average over a small volume that contains an integer number of charge-density-wave periods. This makes the Langevin force in Eq. (2.3a) drop out and we obtain

$$\partial_t \boldsymbol{v}(\boldsymbol{x},t) = \frac{-c_0^2}{\rho_0}\boldsymbol{\nabla}\, n(\boldsymbol{x},t) + \frac{\boldsymbol{q}}{q^2}\omega_0^2(\boldsymbol{\nabla})u(\boldsymbol{x},t) - \frac{e}{m_e}\boldsymbol{E}(\boldsymbol{x},t). \tag{2.5a}$$

Here we have used the identities

$$\frac{\delta H}{\delta\rho(\boldsymbol{x})} = \int d\boldsymbol{y}\left[\frac{\delta H}{\delta u(\boldsymbol{y})}\frac{\delta u(\boldsymbol{y})}{\delta\rho(\boldsymbol{x})} + \frac{\delta H}{\delta n(\boldsymbol{y})}\frac{\delta n(\boldsymbol{y})}{\delta\rho(\boldsymbol{x})}\right],$$

with H from Eq. (2.4b) and

$$\delta n(\boldsymbol{y})/\delta\rho(\boldsymbol{x}) = \delta(\boldsymbol{x}-\boldsymbol{y}),$$

and defined an operator

$$\omega_0^2(\boldsymbol{\nabla}) = \gamma_0[-c_x\partial_x^2 + c_\perp\boldsymbol{\nabla}_\perp^4/q^2] \tag{2.5b}$$

with $\gamma_0 = 2N_F q^2/\rho_0$. For later reference we also define

$$\omega_p^2(\boldsymbol{\nabla}) = 2^{d-1}\pi\frac{\rho_0 e^2}{m_e^2}(-\boldsymbol{\nabla}^2)^{(3-d)/2} - c_0^2\boldsymbol{\nabla}^2. \tag{2.5c}$$

Course graining the continuity equation and using Eq. (2.4a), yields

$$\partial_t n(\boldsymbol{x},t) = -\rho_0\,\boldsymbol{\nabla}\cdot\boldsymbol{v}(\boldsymbol{x},t). \tag{2.6a}$$

Also from the continuity equation, by multiplying by $\sin(\boldsymbol{q}\cdot\boldsymbol{x})$ and coarse graining, we find

$$\partial_t u(\boldsymbol{x},t) = -\boldsymbol{q}\cdot\boldsymbol{v}(\boldsymbol{x},t). \tag{2.6b}$$

Finally, from Eqs. (2.3c) and (2.3d) we obtain, after coarse graining,

$$\boldsymbol{\nabla}^2\boldsymbol{E}(\boldsymbol{x},t) = -2^{d-1}\pi e(-\boldsymbol{\nabla}^2)^{(3-d)/2}n(\boldsymbol{x},t). \tag{2.7}$$

We next derive an equation that couples n and u by taking the divergence of Eq. (2.5a), and using Eqs. (2.3c) and (2.6a),

$$\partial_t^2 n(\boldsymbol{x},t) = -\omega_{\rm p}^2(\boldsymbol{\nabla})n(\boldsymbol{x},t) - \frac{\rho_0}{q^2}(\boldsymbol{q}\cdot\boldsymbol{\nabla})\omega_0^2(\boldsymbol{\nabla})u(\boldsymbol{x},t). \tag{2.8a}$$

A second equation coupling n and u is obtained from Eq. (2.6b) with the help of Eqs. (2.5a) and (2.7), viz.,

$$\partial_t^2 \boldsymbol{\nabla}^2 u(\boldsymbol{x},t) = -\boldsymbol{\nabla}^2\omega_0^2(\boldsymbol{\nabla})u(\boldsymbol{x},t) - \frac{q}{\rho_0}\omega_{\rm p}^2(\boldsymbol{\nabla})\partial_t n(\boldsymbol{x},t). \tag{2.8b}$$

Equation (2.8) constitutes a closed system of partial differential equations for the two dynamical variables n and u. With

$$\omega_0^2(\boldsymbol{k}) = \gamma_0[c_x k_x^2 + c_\perp \boldsymbol{k}_\perp^4/q^2] \tag{2.9a}$$

the Fourier transform of Eq. (2.5b) and

$$\omega_{\rm p}^2(\boldsymbol{k}) = 2^{d-1}\pi\frac{\rho_0 e^2}{m_{\rm e}^2}|\boldsymbol{k}|^{3-d} + c_0^2\boldsymbol{k}^2 \tag{2.9b}$$

the plasma frequency squared, the two resonance frequencies are

$$\Omega_{\rm p}^2(\boldsymbol{k}) = \omega_{\rm p}^2(\boldsymbol{k}) + \omega_0^2(\boldsymbol{k})k_x^2/\boldsymbol{k}^2, \tag{2.10a}$$

$$\Omega_{\rm s}^2(\boldsymbol{k}) = \omega_0^2(\boldsymbol{k})\boldsymbol{k}_\perp^2/\boldsymbol{k}^2. \tag{2.10b}$$

For neutral systems ($e=0$), $\omega_{\rm p}(\boldsymbol{k})$ correctly reduces to the frequency of (first) sound, $c_0|\boldsymbol{k}|$. Switching back to an imaginary time/frequency representation, the corresponding eigenvectors, i.e., the soft modes, are

$$\pi(k) = n(k) + i\frac{\rho_0}{qc_0^2}\omega_0^2(\boldsymbol{k})\frac{k_x}{\boldsymbol{k}^2}u(k), \tag{2.11a}$$

$$\sigma(k) = u(k) + i\frac{q}{\rho_0}\frac{k_x}{\boldsymbol{k}^2}n(k). \tag{2.11b}$$

The σ-σ correlation function or smecton susceptibility then is

$$\chi_{\sigma\sigma}(k) = \frac{\gamma_0 q^2}{N_{\rm F}k_{\rm F}^2}\frac{1}{\Omega_{\rm s}^2(\boldsymbol{k}) - (i\Omega)^2}. \tag{2.12}$$

These results are exactly analogous to the classical case,[3,4] see Eq. (1.1), except that the charged nature of the electron system modifies first sound into a plasma mode. Note the strongly anisotropic wave-vector dependence of both resonance frequencies and of $\Omega_{\rm s}$, in particular. In a classical context, the two modes are usually referred to as first and second sound, respectively, for neutral systems, and as plasma oscillations and second sound for charged ones. In a quantum context, the quanta of first sound and plasma oscillations are usually referred to as phonons and plasmons, respectively, and by analogy we call the quanta of second sound "smectons." The smecton is the Goldstone mode related to the unidirectional charge-density-wave order. It is precisely analogous to the corresponding soft modes in both smectic and cholesteric liquid crystals. By contrast, the helimagnon (the Goldstone mode in a helical magnet), is *not* completely analogous to the classical cholesteric Goldstone mode; it is missing the factor $\boldsymbol{k}_\perp^2/\boldsymbol{k}^2$ in the analog of Eq. (2.10b).[17] This factor is also missing in the phenomenological description of an electronic smectic in Ref. 21, which did not take into account the coupling to plasmons.[28] While this omission makes a large difference for the angular dependence of the resonance frequency, it is of no consequence for the leading temperature dependencies of various observables that couple to the smectons. This is because, as we will see, the latter is determined by a region in wave-vector space where $k_x \sim \boldsymbol{k}_\perp^2 \sim T$ in a scaling sense[29] and in this regime the prefactor is equal to unity to leading order as $T \to 0$.

III. OBSERVABLE CONSEQUENCES OF THE SMECTONS

A. Specific heat

A result that can be immediately obtained from the resonance frequency alone is the smecton contribution to the internal energy U and hence to the specific heat $C = \partial U/V\partial T$. The former is given by $U_{\rm s} = \Sigma_{\boldsymbol{k}}\Omega_{\rm s}(\boldsymbol{k})n_{\rm B}[\Omega_{\rm s}(\boldsymbol{k})]$, with $n_{\rm B}$ the Bose distribution function. The result in 3-d is $C(T\to 0) \propto T^2$ and in 2-d we find

$$C_{\rm s}(T\to 0) = A_C q^2 (T/T_q)^{3/2}. \tag{3.1}$$

Here we have defined a temperature scale $T_q = \sqrt{\gamma_0}\lambda q^2/k_{\rm F}^2$ and A_C is a number of $O(1)$. This agrees with the result obtained in Ref. 21. An inspection of the integral shows that the dominant contribution comes from the region in wave-vector space mentioned at the end of the last section. The resulting temperature dependence is nonanalytic but subleading compared to the Fermi-liquid contribution $C_{\rm FL} \propto T$.

B. Effective quasiparticle theory

In order to consider the effects of the smectons on other observables it is useful to derive an effective action for quasiparticles in the presence of smectic order, in analogy to the theory for helical magnets developed in Ref. 30. In the present case, the effective action takes the form

$$S_{\rm eff}[\bar\psi,\psi] = S_0[\bar\psi,\psi] + \frac{1}{2}\Gamma^2\int dxdy n_q(x)\chi(x,y)n_q(y). \tag{3.2a}$$

Here $\bar\psi$ and ψ are fermion fields, Γ is the interaction amplitude mentioned in the context of Eq. (2.2) above, and $n_q(x)$ is the electron number density $\bar\psi(x)\psi(x)$ with the understanding that n_q contains only wave vectors close to $\boldsymbol{q}$ or $-\boldsymbol{q}$. S_0 is given by

$$S_0[\bar\psi,\psi] = \tilde S_0[\bar\psi,\psi] + \lambda\int dx\,\cos(\boldsymbol{q}\cdot\boldsymbol{x})n_q(x), \tag{3.2b}$$

and $\tilde S_0$ describes free or band electrons plus any interactions in channels other than the one mediated by Γ. χ is the density susceptibility in the relevant wave-vector region, which is dominated by the phase-phase susceptibility χ_{uu}, which in turn, as far as leading hydrodynamic effects are concerned, is the same as the smecton susceptibility, Eq. (2.12),

$$\chi(x,y) \approx \Delta^2 \sin(\boldsymbol{q}\cdot\boldsymbol{x})\sin(\boldsymbol{q}\cdot\boldsymbol{y})\chi_{\sigma\sigma}(x-y). \quad (3.2c)$$

The physical interpretation of $S_{\rm eff}$ is that S_0 contains the smectic order in a mean-field approximation whereas the second term on the right-hand side of Eq. (3.2a) takes into account fluctuations that can be described as an exchange of smectons between electrons.

We now define $\psi_\pm(p) \equiv \psi(p \pm q)$ and $\bar{\psi}_\pm(p) \equiv \bar{\psi}(p \pm q)$. If we use a nearly-free-electron model for $\tilde{S}_0$,[31] this allows us to write

$$S_{\rm eff}[\bar{\psi},\psi] = S_0[\bar{\psi},\psi] + S_{\rm int}[\bar{\psi},\psi], \quad (3.3a)$$

where

$$S_0[\bar{\psi},\psi] = \sum_p \sum_{\sigma=\pm} G_\sigma^{-1}(p)\bar{\psi}_\sigma(p)\psi_\sigma(p) + \lambda\sum_p [\bar{\psi}_+(p)\psi_-(p+q) + \bar{\psi}_-(p)\psi_+(p-q)], \quad (3.3b)$$

$$S_{\rm int}[\bar{\psi},\psi] = -\frac{\lambda^2}{2}\frac{T}{V}\sum_k \chi_{\sigma\sigma}(k)[\delta n_{+-}(k-q) - \delta n_{-+}(k+q)] \times [\delta n_{+-}(-k-q) - \delta n_{-+}(-k+q)]. \quad (3.3c)$$

Here

$$G_\pm^{-1}(p) = i\omega - \xi_{p\pm q}, \quad (3.3d)$$

with $i\omega$ a fermionic Matsubara frequency. $\xi_k = \epsilon_k - \mu$ with μ the chemical potential and ϵ_k the single-fermion energy-momentum relation. We also have defined

$$\delta n_{\sigma_1\sigma_2}(p) = n_{\sigma_1\sigma_2}(p) - \langle n_{\sigma_1\sigma_2}(p)\rangle, \quad (3.3e)$$

where $n_{\sigma_1\sigma_2}(p) = (T/V)\Sigma_p \bar{\psi}_{\sigma_1}(p)\psi_{\sigma_2}(p-k)$. In Eq. (3.3c) we have dropped contributions where the $\chi_{\sigma\sigma}$ appears at wave vectors $\boldsymbol{k} \pm 2\boldsymbol{q}$, as $\chi_{\sigma\sigma}$ is soft only at $k=0$.

The action S_0 can now be diagonalized by a canonical transformation to quasiparticle fields $\bar{\eta}$ and η via

$$\psi_-(p) = [\eta_+(p-q) - \alpha_{p-q}\eta_-(p-q)]/\sqrt{1+\alpha_{p-q}^2},$$

$$\psi_+(p) = [\eta_-(p) + \alpha_p\eta_+(p)]/\sqrt{1+\alpha_p^2}, \quad (3.4a)$$

with

$$\alpha_p = \frac{-1}{2\lambda}[\xi_{p+q} - \xi_p + \sqrt{(\xi_{p+q}-\xi_p)^2 + 4\lambda^2}], \quad (3.4b)$$

and the same relation between $\bar{\psi}_\pm$ and $\bar{\eta}_\pm$. The resulting quasiparticle action is very similar to the one for helimagnets derived in Ref. 30. The main difference is that here the bare quasiparticle Green's function G, Eq. (3.3d), depends on the Stoner-band index σ, where as in the helimagnon case it does not. The action can be written as

$$S[\eta,\bar{\eta}] = S_0[\eta,\bar{\eta}] + S_{\rm int}[\eta,\bar{\eta}], \quad (3.5a)$$

where

$$S_0 = \sum_{p,\sigma}[i\omega - \omega_\sigma(\boldsymbol{p})]\bar{\eta}_\sigma(p)\eta(p), \quad (3.5b)$$

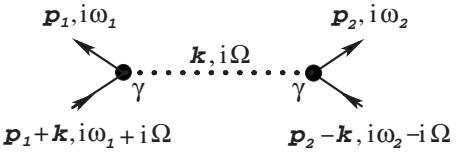

FIG. 1. The effective quasiparticle interaction, Eq. (3.5c), due to smecton exchange. Note that the vertices γ depend on the quasiparticle momenta in addition to the smecton momentum, see Eq. (3.5f).

$$S_{\rm int} = -V_0\frac{T}{V}\sum_k \chi_{\sigma\sigma}(k)\delta d(k)\delta d(-k). \quad (3.5c)$$

Here $V_0 = \lambda^2 q^2/8m_e^2$ and the resonance frequencies $\omega_\pm(\boldsymbol{p})$ are given by

$$\omega_\pm(\boldsymbol{p}) = \frac{1}{2}[\xi_{p+q} + \xi_p \pm \sqrt{(\xi_{p+q}-\xi_p)^2 + 4\lambda^2}]. \quad (3.5d)$$

$\delta d(k) = d(k) - \langle d(k)\rangle$, with

$$d(k) = \sum_p \gamma(\boldsymbol{k},\boldsymbol{p})\sum_\sigma \bar{\eta}_\sigma(p)\eta_\sigma(p-k), \quad (3.5e)$$

$$\gamma(\boldsymbol{k},\boldsymbol{p}) = \frac{2m_e}{q}\frac{\alpha_p - \alpha_{p-k}}{\sqrt{1+\alpha_p^2}\sqrt{1+\alpha_{p-k}^2}}. \quad (3.5f)$$

Note that $\gamma(\boldsymbol{k}\to 0,\boldsymbol{p}) \to 0$. This reflects the fact that the effective interaction described by $S_{\rm int}$ in Eq. (3.5c) represents the coupling of electronic density fluctuations to the phase of the smecton. The phase had no physical significance by itself and the coupling must therefore be to the gradient of the phase. The structure of our effective theory reflects this. The effective interaction is graphically represented in Fig. 1.

C. Quasiparticle relaxation time

The effective theory defined by Eq. (3.5) can now be used to calculate the properties of the quasiparticles by standard means. Electron correlation functions (which ultimately determine observables such as the conductivity) can be recovered from quasiparticle ones by means of the transformation (3.4a). We will first focus on the quasiparticle relaxation time and then briefly discuss transport properties.

The elastic quasiparticle relaxation time $\tau_{\rm el}$ can be obtained by adding quenched disorder to the action and calculating the disorder contribution to the quasiparticle self-energy. It does not qualitatively depend on the dimensionality and the results given for the 3-d helimagnon case in Ref. 30 apply here as well. The inelastic quasiparticle relaxation rate $1/\tau$, which is given by the imaginary part of the quasiparticle self-energy due to the interaction $S_{\rm int}$, see Fig. 2, in $d=2$ is more interesting. To linear order in the effective interaction, and keeping only the leading temperature dependence, the rate averaged over the Fermi surface given by $\omega_+(\boldsymbol{k})=0$ can be written as

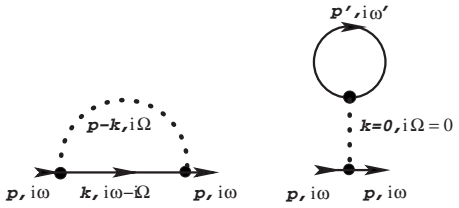

FIG. 2. (a) The Fock or exchange and (b) Hartree or direct contributions to the quasiparticle self-energy due to the effective interaction mediated by smectons. Only diagram (a) contributes to the relaxation rate, as diagram (b) is purely real.

$$\frac{1}{\tau} = V_0 \frac{2\gamma_0 m_e}{N_F k_F^2} \frac{1}{V} \sum_{\boldsymbol{p}} \frac{|p_\perp|}{\omega_0(\boldsymbol{p}) \sinh[\omega_0(\boldsymbol{p})/T]} \times \frac{1}{V} \sum_{\boldsymbol{k}} \frac{(\partial \alpha_{\boldsymbol{k}}/\partial k_\perp)^2}{(1+\alpha_{\boldsymbol{k}})^2} \delta[\omega_+(\boldsymbol{k})] \delta\left[\frac{\partial \omega_+(\boldsymbol{k})}{\partial k_\perp}\right]. \quad (3.6)$$

The integrals over $\boldsymbol{p}$ and $\boldsymbol{k}$ decouple and the temperature dependence of $1/\tau$ comes from the former. Using Eq. (2.9a) we see that it is linear in T, with a logarithmically infinite prefactor. The latter results from the $i\Omega=0$ contribution to the underlying Matsubara frequency sum; the remainder of the sum leads to a $T \ln T$ behavior. This divergence is cut off by a variety of effects that have been neglected in the above treatment. For instance, the underlying lattice structure of a solid (as opposed to a liquid crystal) breaks the rotational symmetry and produces a term $\propto \boldsymbol{k}_\perp^2$ in Eq. (2.9a), which leads to a T^2 behavior of $1/\tau$ at asymptotically low temperatures. Also, screening of the quasiparticle interaction effectively leads to the same result.[32] There thus is a temperature scale that cuts off the logarithmic infinity, leaving a $T \ln T$ in the temperature regime where the current treatment is valid. The remaining question is whether the prefactor given by the $\boldsymbol{k}$ integral in Eq. (3.6) is nonzero. In general it is but this depends on the detailed structure of the Fermi surface. For instance, for underlying nematic order in $d=2$, which is relevant for stripe order, it is easy to see that the $\boldsymbol{k}$ integral is nonzero if the axes of the elliptical Fermi surface are not aligned with $\boldsymbol{q}$ but vanishes if they are.[33] We thus conclude that in a 2-d electronic smectic-C system the quasiparticle relaxation rate displays non-Fermi-liquid behavior and its temperature dependence is

$$1/\tau \propto T(\ln T + \text{const}) \quad (\text{clean}, d=2). \quad (3.7)$$

The prefactor depends on the value of q and, in particular, is strongly dependent on whether qk_F/m_e is small or large compared to λ. This will be discussed in a future publication. In $d=3$ the corresponding temperature dependence is $T^{3/2}$, as it is for helimagnons.[22] For real systems, these results hold in a preasymptotic temperature regime whose size depends on detailed parameter values and it will cross over to a different behavior in the true asymptotic low-temperature regime. Comparing with Ref. 21, who found a stronger behavior $1/\tau \propto \ln T$, we see that the discrepancy stems from the coupling of the smectons to the quasiparticles. In Ref. 21 the quasiparticles couple to the smecton phase, rather than to its gradient as they should on physical grounds.

The presence of quenched disorder modifies the above considerations. In the weak-disorder regime,[30,34] $T \gg \lambda/(\epsilon_F \tau_{el})^2$, the calculation proceeds in analogy to Ref. 30. The result is

$$1/\tau \propto T^{1/2} \quad (\text{weak disorder}, d=2) \quad (3.8)$$

in $d=2$ and $1/\tau \propto T$ in $d=3$.

D. Transport coefficients

The above results pertain to the quasiparticle relaxation time, which is not easy to observe directly. Various transport coefficients depend on relaxation times that are generally different from the quasiparticle one and that are much harder to calculate. For instance, the Boltzmann equation for the electrical conductivity σ, with the scattering treated in Born approximation, leads to a transport relaxation rate that is, for the current problem, weaker by one power of temperature than the quasiparticle rate.[22] Technically, the electrical transport relaxation rate is given by Eq. (3.6) with an additional factor proportional to $\boldsymbol{p}_\perp^2$ in the integrand of the $\boldsymbol{p}$ integral. This leads to $\sigma \propto 1/T^2$ in $d=2$ and $\sigma \propto 1/T^{5/2}$ in $d=3$.

The situation is different, however, for the thermal conductivity κ. The temperature dependence of κ/T is given by the quasiparticle relaxation rate.[35,36] The physical reason is that an electric current can relax only by the electrons changing direction since the electron's electric charge is conserved. In the calculation of the relaxation time, this leads to a geometric factor that weighs backscattering more strongly than forward scattering and this is manifested in the additional factor of $\boldsymbol{p}_\perp^2$ in the integrand. An electron's energy is not conserved, however, in an inelastic-scattering process, and hence this geometric factor is absent in the calculation of the leading temperature dependence of the thermal transport coefficient.[35,37] In the current problem, this leads to

$$\kappa/T \propto 1/T \ln T \quad (\text{clean}, d=2) \quad (3.9)$$

for clean systems in $d=2$. In $d=3$, the corresponding temperature dependence is $T^{-3/2}$. The Wiedemann-Franz law is thus violated, as it is in the case of electron-phonon scattering, and the Lorenz ratio, defined by $L=\kappa/T\sigma$, is proportional to $T \ln T$. In weakly disordered systems, the temperature dependence of κ/T is governed by Eq. (3.8) but one needs to take into account the residual value κ_r of the thermal conductivity. Since the residual values of the transport coefficients are determined by elastic-scattering processes, the latter is related to the residual electrical conductivity σ_r by the Wiedemann-Franz law $\kappa_r/T\sigma_r = L_r$ with the Lorenz ratio $L_r = \pi^2 k_B^2/3e^2$ as a constant.

IV. SUMMARY AND CONCLUSION

In summary, we have determined the Goldstone modes and their properties in electronic smectics or stripe phases. In an isotropic model system the soft modes (smectons) are precisely analogous to those in both smectic and cholesteric classical liquid crystals. Their wave-vector dependence dif-

fers (albeit not in a scaling sense) from that of the helimagnons in helical magnets, which are analogous to classical cholesteric liquid crystals. This difference is due to the fact that spin dynamics are different from density dynamics.

In $d=2$, the smectons contribute a term proportional to $T^{3/2}$ to the specific heat and the quasiparticle relaxation rate $1/\tau$ as well as T/κ, with κ the heat conductivity, are proportional to $T \ln T$ in clean systems. In weakly disordered systems, the corresponding leading temperature dependence is given by $T^{1/2}$. In $d=3$, the corresponding temperature dependencies are T^2 (for the specific heat), $T^{3/2}$ (for the relaxation rate and T/κ in clean systems), and T (for the relaxation rate and T/κ in the weak-disorder regime), respectively. In the weak-disorder regime, the leading temperature dependencies of $1/\tau$, $1/\sigma$, and T/κ. The leading temperature dependence of the electrical resistivity $1/\sigma$ is weaker than that of $1/\tau$ by one power of T in clean systems, and the same as that of $1/\tau$ in weakly disordered ones, respectively. Qualititively, all of these results also hold for the exchange of helimagnons between electrons in helical magnet,[22,38] and they are summarized in Table I.

We conclude with some speculations pertaining to the electrical conductivity. While the standard weak-coupling treatment of the Boltzmann equation yields a resistivity $\rho \propto T^2$ in 2-d clean systems as mentioned above, it is conceivable that in a strongly correlated electron system mode-mode coupling effects mix the various time scales and lead to a single relaxation time. It is currently not known whether this hypothesis is correct, or what it takes at a technical level to demonstrate it, but it provides a possible mechanism for producing an electrical resistivity that is linear in T in 2-d or quasi-2-d systems.

TABLE I. Leading smecton contributions to the temperature dependencies of the specific heat (C), the quasiparticle relaxation rate ($1/\tau$), the heat conductivity (κ), and the electrical conductivity (σ). The same results hold for the helimagnon contributions in helical magnets. See the text for additional information.

	Clean		Weak disorder	
	$d=2$	$d=3$	$d=2$	$d=3$
C	$T^{3/2}$	T^2	$T^{3/2}$	T^2
$1/\tau$	$T \ln T$	$T^{3/2}$	$T^{1/2}$	T
T/κ	$T \ln T$	$T^{3/2}$	$T^{1/2}$	T
$1/\sigma$	T^2	$T^{5/2}$	$T^{1/2}$	T

ACKNOWLEDGMENT

This work was supported by the NSF under Grants No. DMR-05-29966 and No. DMR-05-30314.

[1] P. G. DeGennes and J. Prost, *The Physics of Liquid Crystals* (Clarendon, Oxford, 1993).
[2] P. Chaikin and T. C. Lubensky, *Principles of Condensed Matter Physics* (Cambridge University, Cambridge, 1995).
[3] T. C. Lubensky, Phys. Rev. A **6**, 452 (1972).
[4] P. C. Martin, O. Parodi, and P. S. Pershan, Phys. Rev. A **6**, 2401 (1972).
[5] M. M. Fogler, A. A. Koulakov, and B. I. Shklovskii, Phys. Rev. B **54**, 1853 (1996).
[6] R. Moessner and J. T. Chalker, Phys. Rev. B **54**, 5006 (1996).
[7] M. P. Lilly, K. B. Cooper, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **82**, 394 (1999).
[8] S. A. Kivelson, E. Fradkin, and V. J. Emery, Nature (London) **393**, 550 (1998).
[9] B. Binz, A. Vishwanath, and V. Aji, Phys. Rev. Lett. **96**, 207202 (2006).
[10] I. Fischer, N. Shah, and A. Rosch, Phys. Rev. B **77**, 024415 (2008).
[11] S. Tewari, D. Belitz, and T. R. Kirkpatrick, Phys. Rev. Lett. **96**, 047207 (2006).
[12] V. Oganesyan, S. A. Kivelson, and E. Fradkin, Phys. Rev. B **64**, 195109 (2001).
[13] I. I. Pomeranchuk, Zh. Eksp. Teor. Fiz. **35**, 524 (1958) [Sov. Phys. JETP **8**, 361 (1959)].
[14] E. Fradkin and S. A. Kivelson, Phys. Rev. B **59**, 8065 (1999).
[15] S. A. Kivelson, I. P. Bindloss, E. Fradkin, V. Oganesyan, J. M. Tranquada, A. Kapitulnik, and C. Howald, Rev. Mod. Phys. **75**, 1201 (2003).
[16] Y. Ishikawa, K. Tajima, D. Bloch, and M. Roth, Solid State Commun. **19**, 525 (1976).
[17] D. Belitz, T. R. Kirkpatrick, and A. Rosch, Phys. Rev. B **73**, 054431 (2006).
[18] M. M. Fogler and V. M. Vinokur, Phys. Rev. Lett. **84**, 5828 (2000).
[19] A. Lopatnikova, S. H. Simon, B. I. Halperin, and X.-G. Wen, Phys. Rev. B **64**, 155301 (2001).
[20] C. Wexler and A. T. Dorsey, Phys. Rev. B **64**, 115312 (2001).
[21] K. Sun, B. M. Fregoso, M. J. Lawler, and E. Fradkin, Phys. Rev. B **78**, 085124 (2008).
[22] D. Belitz, T. R. Kirkpatrick, and A. Rosch, Phys. Rev. B **74**, 024409 (2006); **76**, 149902(E) (2007).
[23] Here we neglect higher harmonics, which will appear in the saddle-point solution for any reasonable model Hamiltonian. This is the same approximation that is usually made in classical liquid crystals, see Ref. 2.
[24] S.-K. Ma, *Modern Theory of Critical Phenomena* (Benjamin, Reading, MA, 1976).
[25] We neglect viscosity terms that lead to damping of the soft modes. The damping effects are rather complicated, as they are in classical liquid crystals,[3] and will be discusssed in a separate publication.
[26] Equation (2.3c) is valid for $d=3,2$. For $d=3$, it is the usual Poisson equation. For $d=2$, it reflects that fact that the electrons are confined to 2-d while the electromagnetic field lines still extend into 3-d space, see Ref. 39. We only consider systems in zero magnetic field.
[27] P. C. Hohenberg and B. I. Halperin, Rev. Mod. Phys. **49**, 435 (1977).

[28] One might ask how a massive mode (the plasmon) can modify a soft mode by coupling to it. The answer lies in the fact that the modification [namely, the factor of $\boldsymbol{k}_\perp^2/\boldsymbol{k}^2$ in Eq. (2.10b)], although it introduces a strong angular dependence of the resonance frequency, is of $O(1)$ in a scaling sense, and hence cannot be ruled out by power counting.

[29] In our notation, $a \propto b$ and $as \sim b$ mean "a is proportional to b" and "a scales as b," respectively.

[30] T. R. Kirkpatrick, D. Belitz, and R. Saha, Phys. Rev. B **78**, 094407 (2008).

[31] We note that all interactions that are necessary to create and stabilize smectic order are already implicit in the second contribution to the action S_0, Eq. (3.2b), i.e., in the existence of a nonzero Stoner gap λ. The residual interactions included in $\tilde{S}_0$ are not expected to have a qualitative influence on our results and we neglect them.

[32] T. R. Kirkpatrick and D. Belitz (unpublished).

[33] In $d=3$ the low-T behavior of $1/\tau$ is known to depend on the functional form of $\xi_{\boldsymbol{k}}$, see Ref. 22. In $d=2$ this is not the case and the results given hold both for a nearly-free-electron model and for $\xi_{\boldsymbol{k}}$ that reflect a coupling to the underlying lattice.

[34] G. Zala, B. N. Narozhny, and I. L. Aleiner, Phys. Rev. B **64**, 214204 (2001).

[35] J. M. Ziman, *Principles of the Theory of Solids* (Cambridge University Press, Cambridge, 1972).

[36] A. H. Wilson, *The Theory of Metals* (Cambridge University Press, Cambridge, 1954).

[37] N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Holt, Rinehart and Winston, New York, 1976).

[38] T. R. Kirkpatrick, D. Belitz, and R. Saha, Phys. Rev. B **78**, 094408 (2008).

[39] F. Stern, Phys. Rev. Lett. **18**, 546 (1967).